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THE USE OF MARKOV RENEWAL THEORY IN RURAL WATER SUPPLIES

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1. Introduction:

Water has quantitative, qualitative, spatial, temporal and state dimensions. The objective of a rural potable water supply system is to satisfy the demands for water delivered in required amounts at times and places desired by users. Stated this way the objective of the system cannot be met acceptably by decreasing demand. Demand is encouraged to fall during periods of short age, but this is not regarded as an acceptable solution for imbalances.

There are alternative paths to system objectives - new sources, (traditional and untraditional ones), recycling and reuse, spill capture, increased production from old sources, runoff in the service area, and network redesign. A revealing dichotomy is apparent - excepting the first, each of the approaches refers to water already flowing within the network itself, or in its area of geographic occupance. The significance of this is that the latter solutions may make it unnecessary to tap distant sources, thus reducing the level of inter-rural area competition for dwindling available watersheds.

2. Linked Trip:

In rural areas, a designer is subjected to varieties of water supply system and the performance of the system also varies according to geographical location. Three dimensions characterize linked trip making behaviour. The first of these is the type of water supply system (i.e., new sources - traditional and unconventional ones, recycling and reuse, spill capture, increased production from old sources, runoff in the service area and network design), the second is the system performance (supply - demand) in the area. The time spent by the system at the area forms the second dimension. The spatial location of the area forms the third dimension of the linked trip.

In order to understand how these dimensions form the structure of rural linked trips, it is first necessary to consider a system of states or conditions into which a designer might enter. Each state is defined by two parameters: a geographical location and the type of water supply he has to provide in the area.

Geographical locations may be designated by dividing a region into zones and numbering these zones. Likewise, type of supply may be defined and given numbers. Then if r refers to the type of supply, and s indicates zone s, in which the type of supply is located, Y_{rs} defines a state. One may think of a state as being uniquely defined by combinations of measures along the type of supply and spatial dimension.

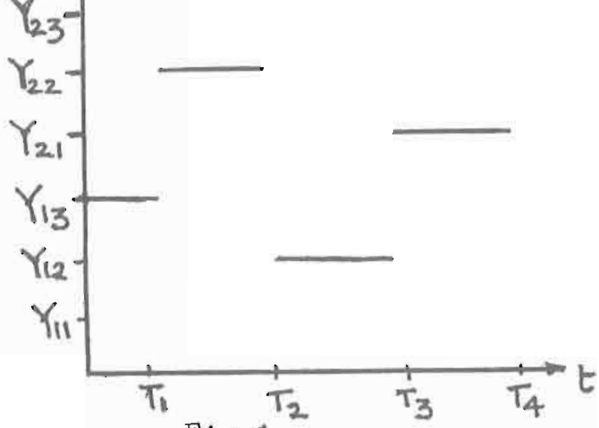

Fig-1.

Fig-1 shows one possible realization of a linked trip journey through a system of states Y_{rs}. In this example, a system of three geographical zones and two types of supply is considered, thus there are only six possible states.

It is possible that not all types of supply would be found in every zone; thus it is expected that the number of states would be less than r x s. At time T_1 the designer moves from state Y_{13} to state Y_{22}. Then after remaining in Y_{22} for $(T_2 - T_1)$ time units, he again moves, this time to state Y_{12}. After $(T_3 - T_2)$ time units he makes a transition to state Y_{21}, and so forth.

Although Fig-1 shows the journey of only one designer, it does point to two processes which together determine the structure of linked trips. On one hand, the designer has chosen to visit the sequence of states (Y_{13}, Y_{22}, Y_{12}, Y_{21},). At the same time he has decided to remain in these zones the lengths of time T_1, $(T_2 - T_1)$, $(T_3 - T_2)$ and $(T_4 - T_3)$. His journey may therefore be viewed as two simultaneous series of design decisions; where to go next and how long to stay in each state. Furthermore, because these processes are not predictable in a deterministic manner, they must be considered to be two simultaneous stochastic processes.

3. Markov-Renewal Process

It is a stochastic process in which the choice of states follows a Markov chain but the time spent in any state is a random quantity depending upon both that state and perhaps the next state to be visited.

This theory extensively developed by Cinlar (1969) essentially combines two stochastic processes; a Markov process and a Renewal process. In applying Markov Renewal theory to the linked trip making process, two basic assumptions must be made. First it must be assumed that the choice of states, $(X(n);\ n = 0, 1, 2, \ldots)$ forms a time homogeneous Markov chain. Second, the travel time between any two states is assumed to comprise part of the so-journ time in the origin state. This assumption is necessary in order to view designers as making instantaneous transitions between states. Thus, designers are always considered to be within a state, and no artificial states need to be constructed to accomodate designers in transit.

4. Transition Probabilities

The first aspect of linked trip making which is of interest is the probability of a trip between any given pair of states i, j.

Let

$F_{i,j}(t)$ = Distribution function of times spent in state i for designer who will next visit to state j.

Also, let

$\tilde{A}_{ij}$ = Probability of going next to state j from state i.

Then,

$$A_{ij}(t) = \tilde{A}_{ij} \cdot F_{ij}(t) \quad \ldots (1)$$

= Probability of going in one transition from state i to state j within the time interval (0, t).

5. So-journ time

The time spent in state j for those designers going next to state k is $(T_{n+1} - T_n / X_n = j, X_{n+1} = k)$.

Let

U_{jk} = Expected length of stay in j for those going next to k.

Therefore,

$$U_{jk} = E(T_{n+1} - T_n / X_n = j, X_{n+1} = k)$$

or

$$U_{jk} = \int_0^\infty t \cdot f_{jk}(t)\, dt \quad \ldots (2)$$

where $f_{jk}(t)$ = density function

$$= d/dt\ F_{jk}\ (t) \ldots (3)$$

6. Number of visits to state

The number of visits to any state j in a time interval (0,t) is of obvious interest in any analysis of linked trip making. This information will be useful in providing the number of designs

with its merits and demerits.

The Markov-Renewal matrix R(t) provides just this information. Specifically, R_{ij} (t) is the number of expected visits that a designer in state i will make to state j in the time interval (0,t), its Laplace transform does this.

$$R^*(s) = I + A^*(s) + (A^*(s)) + . \ldots (4)$$

or,
as the number of states is finite, then

$$R^*(s) = (I - A^*(s))^{-1} \ldots (5)$$

7. Number of transitions

One aspect of linked trip making which may be of interest is the expected number of states visited by a designer during his tenure. That is, how many incomplete designs is he likely to make during a project which begins in state j. This characteristic is an indication of quality requirements in water supply of various types.
Let
M_i (t) = Total number of expected states visited during a project which begins in state i,

$$= \sum_j R_{ij}\ (t) \ldots (6)$$

8. Conclusion

The chief objective in linked trip-making analysis is the prediction of the effects of public policies upon linked trip patterns of "supply-demand". This objective may be accomplished using the Markov-Renewal model by performing two tasks. First, the effects of a policy upon

$\tilde{A}$ and F(t)

must be determined. Second, the resulting effects upon linked trip structures must be predicted and evaluated (Achuthan, 1981).

The numerical part of this model is in progress.

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10. References

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